Quiz 8, Linear Algebra

Fall 2017, Dr. Adam Graham-Squire

4 min

⇒ 20 min in class.

Name: Key

1. (4 points) Suppose the solutions of a homogeneous system of five linear equations in six unknowns are all multiples of one nonzero solution. Will the system necessarily have a solution for every choice of constants on the right sides of the equations? Explain.

$$\begin{array}{cc} & 6 \\ 5 & \left[\begin{array}{c|c} A & \end{array}\right] \end{array}$$

"Solutions are all multiples of one nonzero solution"

means $\dim(\text{Nul } A) = 1$ ✓

$\Rightarrow$ rank $= 5$

$\Rightarrow$ full rank (pivot in every row) ✓

$\Rightarrow$ solution for all $\vec{b}$ of $A\vec{x} = \vec{b}$ ✓

$\boxed{\text{Yes}}$

2. (4 points) Find the characteristic polynomial and the eigenvalues of $A = \begin{bmatrix} 1 & \cancel{4}\,5 \\ 4 & 2 \end{bmatrix}$, if they exist.

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 5 \\ 4 & 2-\lambda \end{bmatrix}$$

$$0 = \det(A - \lambda I) = 2 - 3\lambda + \lambda^2 \overset{-20}{\cancel{+8}}$$

$$0 = \boxed{\lambda^2 - 3\lambda - 18} \Rightarrow \text{characteristic eq.}$$

$$0 = (\lambda - 6)(\lambda + 3)$$

$$\Rightarrow \boxed{\lambda = 6 \quad \text{or} \quad \lambda = -3} \Rightarrow \text{eigenvalues.}$$

3. (2 points) Is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ an eigenvector for the matrix $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$? Is so, find the corresponding eigenvalue. Show/explain your work.

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

and $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ is <u>not</u> a multiple of $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$,

so $\boxed{\text{No}}$